

## Lesson 1: Interpreting Division of a Fraction by a Whole

### Number—Visual Models

#### Classwork

##### Opening Exercise

Draw a model of the fraction.

Describe what the fraction means.

#### Example 1

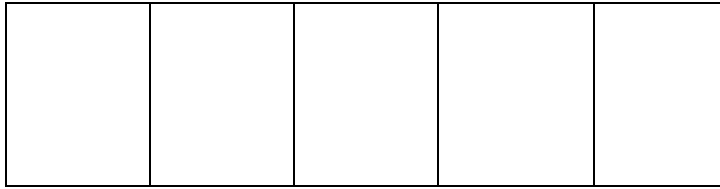
Maria has  $\frac{3}{4}$  lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?

How can this question be modeled?

**Example 2**

Let's look at a slightly different example. Imagine that you have  $\frac{2}{5}$  of a cup of frosting to share equally among three desserts. How would we write this as a division question?

We can start by drawing a model of two-fifths.



How can we show that we are dividing two-fifths into three equal parts?

What does this part represent?

**Exercises 1–5**

For each question below, rewrite the problem as a multiplication question. Then, model the answer.

1.  $\frac{1}{2} \div 6 =$

2.  $\frac{1}{3} \div 3 =$

3.  $\frac{1}{5} \div 4 =$

4.  $\frac{3}{5} \div 4 =$

5.  $\frac{2}{3} \div 4 =$

**Problem Set**

Rewrite each problem as a multiplication question. Model your answer.

1.  $\frac{2}{5} \div 5$

2.  $\frac{3}{4} \div 2$

## Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

### Classwork

#### Example 1

Question # \_\_\_\_\_

Write it as a division question.

\_\_\_\_\_

Write it as a multiplication question.

\_\_\_\_\_

Make a rough draft of a model to represent the question:

As you travel to each model, be sure to answer the following questions:

Original Questions	Write the division question that was answered in each model.	What multiplication question could the model also answer?	Write the question given to each group as a multiplication question.
1. How many $\frac{1}{2}$ miles are in 12 miles?			
2. How many quarter hours are in 5 hours?			
3. How many $\frac{1}{3}$ cups are in 9 cups?			
4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?			
5. How many one-fifths are in 7 wholes?			



2. George bought 12 pizzas for a birthday party. If each person will eat  $\frac{3}{8}$  of a pizza, how many people can George feed with 12 pizzas?
3. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up  $\frac{3}{4}$  of a mile, how many family members are needed to clean the adopted section?



4. Margo is freezing 8 cups of strawberries. If this is  $\frac{2}{3}$  of the total strawberries that were picked, how many cups of strawberries did Margo pick?
5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent  $\frac{5}{8}$  of all the logs that need to be chopped, how many logs need to be chopped in all?

**Problem Set**

Rewrite each problem as a multiplication question. Model your answer.

1. Nicole has used 6 feet of ribbon. This represents  $\frac{3}{8}$  of the total amount of ribbon she started with. How much ribbon did Nicole have at the start?
2. How many quarter hours are in 5 hours?

## Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

### Classwork

#### Opening Exercise

Draw a model to represent  $12 \div 3$ .

How could we reword this question?

#### Example 1

$$\frac{8}{9} \div \frac{2}{9}$$

Draw a model to show the division problem.

**Example 2**

$$\frac{9}{12} \div \frac{3}{12}$$

Be sure to draw a model to support your answer.

**Example 3**

$$\frac{7}{9} \div \frac{3}{9}$$

Be sure to create a model to support your answer.

**Exercises 1–6**

For the following exercises, rewrite the division problem. Then, be sure to draw a model to support your answer.

1. How many fourths are in three fourths?

Draw a model to support your answer.

How are Example 2 and Exercise 1 similar?

How are the divisors and dividends related?

What conclusions can you draw from these observations?

2.  $\frac{4}{5} \div \frac{2}{5}$

3.  $\frac{9}{4} \div \frac{3}{4}$

4.  $\frac{7}{8} \div \frac{2}{8}$

5.  $\frac{13}{10} \div \frac{2}{10}$

6.  $\frac{11}{9} \div \frac{3}{9}$

**Lesson Summary**

When dividing a fraction by a fraction with the same denominator, we can use the general rule  $\frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$ .

**Problem Set**

For the following exercises, rewrite the division problem in words. Then, be sure to draw a model to support your answer.

1.  $\frac{15}{4} \div \frac{3}{4}$

2.  $\frac{8}{5} \div \frac{3}{5}$



## Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

### Classwork

#### Opening Exercise

Write at least three equivalent fractions for each fraction below. Be sure to show how the two fractions are related.

a.  $\frac{2}{3}$

b.  $\frac{10}{12}$

#### Example 1

Molly purchased  $\frac{11}{8}$  cups of strawberries. If she eats  $\frac{2}{8}$  cups per serving, how many servings does Molly have?

Use a model to prove your answer.

**Example 2**

Now imagine that Xavier, Molly's friend, purchased  $\frac{11}{8}$  cups of strawberries. If he eats  $\frac{3}{4}$  cups of strawberries per serving, how many servings will he have? Use a model to prove your answer.

**Example 3**

Find the quotient:  $\frac{3}{4} \div \frac{2}{3}$ . Use a model to show your answer.

**Exercises 1–5**

A model should be included in your solution.

1.  $\frac{6}{2} \div \frac{3}{4}$

2.  $\frac{2}{3} \div \frac{2}{5}$

3.  $\frac{7}{8} \div \frac{1}{2}$

4.  $\frac{3}{5} \div \frac{1}{4}$

5.  $\frac{5}{4} \div \frac{1}{3}$

**Problem Set**

Draw a model to support your answer to the division questions.

1.  $\frac{8}{9} \div \frac{4}{9}$

2.  $\frac{9}{10} \div \frac{4}{10}$

3.  $\frac{3}{5} \div \frac{1}{3}$

4.  $\frac{3}{4} \div \frac{1}{5}$



## Lesson 5: Creating Division Stories

### Classwork

#### Opening Exercises

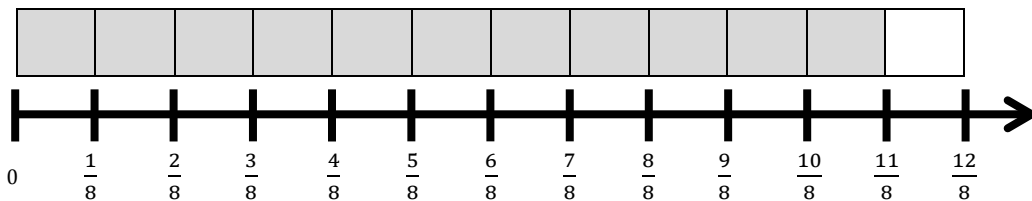
Fraction Bar:

$$\frac{8}{9} \div \frac{2}{9}$$



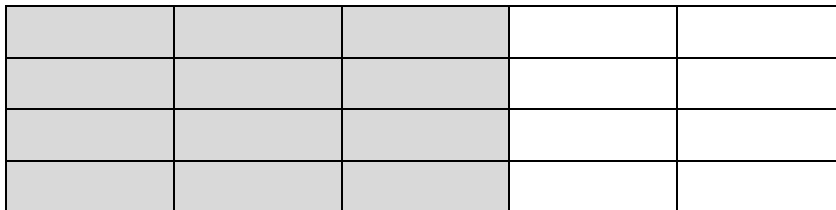
Number Line:

Xavier, Molly’s friend, purchased  $\frac{11}{8}$  cups of strawberries. If he eats  $\frac{3}{4}$  of a cup of strawberries per serving, how many servings will he have?



Area Model:

$$\frac{3}{5} \div \frac{1}{4}$$



**Example 1**

$$\frac{1}{2} \div \frac{1}{8}$$

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation.



**Exercise 1**

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

**Example 2**

$$\frac{3}{4} \div \frac{1}{2}$$

Step 1: Decide on an interpretation.

Step 2: Draw a diagram.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation.

## Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as cups, yards, or miles if you prefer.

### Lesson Summary

The method of creating division stories includes five steps:

Step 1: Decide on an interpretation (measurement or partitive). Today we used measurement division.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation. This means writing a story problem that is interesting, realistic, and short. It may take several attempts before you find a story that works well with the given dividend and divisor.

### Problem Set

Please use each of the five steps of the process you learned. Label each step.

1. Write a measurement division story problem for  $6 \div \frac{3}{4}$ .
2. Write a measurement division story problem for  $\frac{5}{12} \div \frac{1}{6}$ .



## Lesson 6: More Division Stories

### Classwork

#### Example 1

Divide  $50 \div \frac{2}{3}$

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation.

**Exercise 1**

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, dollars, but your situation must be unique. You could try another unit, such as miles, if you prefer.

**Example 2**

Divide  $45 \div \frac{3}{8}$

Step 1: Decide on an interpretation.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation.

### Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. Try a different unit. Remember, spending money gives a “before and after” word problem. If you use dollars, you are looking for a situation where  $\frac{3}{8}$  of some greater dollar amount is \$45.

**Problem Set**

1. Write a partitive division story problem for  $45 \div \frac{3}{5}$ .
2. Write a partitive division story problem for  $100 \div \frac{2}{5}$ .

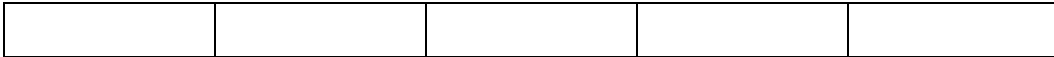


## Lesson 7: The Relationship Between Visual Fraction Models and Equations

### Classwork

#### Example 1

$$\frac{3}{4} \div \frac{2}{5}$$



Shade 2 of the 5 sections  $\left(\frac{2}{5}\right)$ .

Label the part that is known  $\left(\frac{3}{4}\right)$ .

Make notes below on the math sentences needed to solve the problem.

**Example 2**

$$\frac{1}{4} \div \frac{2}{3}$$

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Show the number sentences below.

**Example 3**

$$\frac{2}{3} \div \frac{3}{4}$$

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Show the number sentences below.

**Lesson Summary**

Connecting models of fraction division to multiplication through the use of reciprocals helps in understanding the “invert and multiply” rule.

**Problem Set**

1. Draw a model that shows  $\frac{2}{5} \div \frac{1}{3}$ . Find the answer as well.
2. Draw a model that shows  $\frac{3}{4} \div \frac{1}{2}$ . Find the answer as well.



## Lesson 8: Dividing Fractions and Mixed Numbers

### Classwork

#### Example 1: Introduction to Calculating the Quotient of a Mixed Number and a Fraction

Carli has  $4\frac{1}{2}$  walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has  $\frac{5}{6}$  of a gallon left.

a. How much paint can she use on each wall in order to have enough to paint the remaining walls?

b. Calculate the quotient.

$$\frac{2}{5} \div 3\frac{4}{7}$$

**Exercise**

Show your work for the memory game in the boxes provided below.

A.
B.
C.
D.
E.
F.
G.
H.
I.
J.
K.
L.

**Problem Set**

Calculate each quotient.

1.  $\frac{2}{5} \div 3\frac{1}{10}$

2.  $4\frac{1}{3} \div \frac{4}{7}$

3.  $3\frac{1}{6} \div \frac{9}{10}$

4.  $\frac{5}{8} \div 2\frac{7}{12}$





## Lesson 9: Sums and Differences of Decimals

### Classwork

#### Example 1

$$25\frac{3}{10} + 376\frac{77}{100}$$

#### Example 2

$$426\frac{1}{5} - 275\frac{1}{2}$$

### Exercises 1–5

Calculate each sum or difference.

1. Samantha and her friends are going on a road trip that is  $245\frac{7}{50}$  miles long. They have already driven  $128\frac{53}{100}$ . How much farther do they have to drive?

2. Ben needs to replace two sides of his fence. One side is  $367\frac{9}{100}$  meters long, and the other is  $329\frac{3}{10}$  meters long. How much fence does Ben need to buy?
3. Mike wants to paint his new office with two different colors. If he needs  $4\frac{4}{5}$  gallons of red paint and  $3\frac{1}{10}$  gallons of brown paint, how much paint does he need in total?
4. After Arianna completed some work, she figured she still had  $78\frac{21}{100}$  pictures to paint. If she completed another  $34\frac{23}{25}$  pictures, how many pictures does Arianna still have to paint?

Use a calculator to convert the fractions into decimals before calculating the sum or difference.

5. Rahzel wants to determine how much gasoline he and his wife use in a month. He calculated that he used  $78\frac{1}{3}$  gallons of gas last month. Rahzel's wife used  $41\frac{3}{8}$  gallons of gas last month. How much total gas did Rahzel and his wife use last month? Round your answer to the nearest hundredth.

**Problem Set**

1. Find each sum or difference.

a.  $381\frac{1}{10} - 214\frac{43}{100}$

b.  $32\frac{3}{4} - 12\frac{1}{2}$

c.  $517\frac{37}{50} + 312\frac{3}{100}$

d.  $632\frac{16}{25} + 32\frac{3}{10}$

e.  $421\frac{3}{50} - 212\frac{9}{10}$

2. Use a calculator to find each sum or difference. Round your answer to the nearest hundredth.

a.  $422\frac{3}{7} - 367\frac{5}{9}$

b.  $23\frac{1}{5} + 45\frac{7}{8}$



## Lesson 10: The Distributive Property and the Products of Decimals

### Classwork

#### Opening Exercise

Calculate the product.

a.  $200 \times 32.6$

b.  $500 \times 22.12$

#### Example 1: Introduction to Partial Products

Use partial products and the distributive property to calculate the product.

$200 \times 32.6$

#### Example 2: Introduction to Partial Products

Use partial products and the distributive property to calculate the area of the rectangular patio shown below.



**Exercises**

Use the boxes below to show your work for each station. Make sure that you are putting the solution for each station in the correct box.

Station One:

Station Two:

Station Three:

Station Four:

Station Five:

**Problem Set**

Calculate the product using partial products.

1.  $400 \times 45.2$

2.  $14.9 \times 100$

3.  $200 \times 38.4$

4.  $900 \times 20.7$

5.  $76.2 \times 200$







**Exercises 1–4**

1. Calculate the product:  $324.56 \times 54.82$ .
2. Kevin spends \$11.25 on lunch every week during the school year. If there are 35.5 weeks during the school year, how much does Kevin spend on lunch over the entire school year? Remember to round to the nearest penny.
3. Gunnar's car gets 22.4 miles per gallon, and his gas tank can hold 17.82 gallons of gas. How many miles can Gunnar travel if he uses all of the gas in the gas tank?
4. The principal of East High School wants to buy a new cover for the sand pit used in the long jump competition. He measured the sand pit and found that the length is 29.2 feet and the width is 9.8 feet. What will the area of the new cover be?

**Problem Set**

Solve each problem. Remember to round to the nearest penny when necessary.

1. Calculate the product:  $45.67 \times 32.58$ .
2. Deprina buys a large cup of coffee for \$4.70 on her way to work every day. If there are 24 work days in the month, how much does Deprina spend on coffee throughout the entire month?
3. Krego earns \$2,456.75 every month. He also earns an extra \$4.75 every time he sells a new gym membership. Last month, Krego sold 32 new gym memberships. How much money did Krego earn last month?
4. Kendra just bought a new house and needs to buy new sod for her backyard. If the dimensions of her yard are 24.6 feet by 14.8 feet, what is the area of her yard?



## Lesson 12: Estimating Digits in a Quotient

### Classwork

#### Opening Exercise

Show an example of how you would solve  $5,911 \div 23$ . You can use any method or model to show your work. Just be sure that you can explain how you arrived at your solution.

#### Example 1

We can also use estimates before we divide to help us solve division problems. In this lesson, we will be using estimation to help us divide two numbers using the division algorithm.

Estimate the quotient of  $8,085 \div 33$ . Then, divide.

Create a model to show the division of 8,085 by 33.





4.  $4,732 \div 52$

a. Estimate the quotient.

b. Use the algorithm to divide.

c. Check your work.



**Problem Set**

Complete the following steps for each problem:

- Estimate the quotient.
- Use the division algorithm to solve.
- Show a model that supports your work with the division algorithm.
- Check your work.

1.  $3,312 \div 48$

2.  $3,125 \div 25$

3.  $1,344 \div 14$



## Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

### Classwork

#### Example 1

- Create a model to divide:  $1,755 \div 27$ .
- Use the division algorithm to show  $1,755 \div 27$ .
- Check your work

#### Example 2

Find the quotient of  $205,276 \div 38$ .

**Example 3**

Find the quotient of  $17,216,673 \div 23$ .

**Exercises 1–6**

For each question, you need to do the following:

- Solve the question. Next to each line, explain your work using place value.
- Evaluate the reasonableness of your answer.

1.  $891,156 \div 12$

2.  $484,692 \div 78$

3.  $281,886 \div 33$

4.  $2,295,517 \div 37$

5.  $952,448 \div 112$

6.  $1,823,535 \div 245$

**Problem Set**

1.  $459,054 \div 54$
2.  $820,386 \div 102$
3.  $1,183,578 \div 227$

## Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

### Classwork

#### Example 1

Divide:  $31,218 \div 132$ .

#### Example 2

Divide:  $974.835 \div 12.45$ .





3. Jerod is making candles from beeswax. He has 132.72 ounces of beeswax. If each candle uses 8.4 ounces of beeswax, how many candles can he make? Will there be any wax left over?
4. There are 20.5 cups of batter in the bowl. If each cupcake uses 0.4 cups of batter, how many cupcakes can be made?
5. In Exercises 3 and 4, how were the remainders, or extra parts, interpreted?

6.  $159.12 \div 6.8$

7.  $167.67 \div 8.1$

**Problem Set**

1. Asian purchased 3.5 lb. of his favorite mixture of dried fruits to use in a trail mix. The total cost was \$16.87. How much did the fruit cost per pound?
2. Divide:  $994.14 \div 18.9$ .



## Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

### Classwork

#### Opening Exercises

Start by finding the quotient of 1,728 and 32.

What would happen if we multiplied the divisor by 10?  $1,728 \div 320$

What would happen if we multiplied the dividend by 10?  $17,280 \div 32$

What would happen if we multiplied both the divisor and dividend by 10?  $17,280 \div 320$

What would happen if we multiplied both the divisor and dividend by 100?  $172,800 \div 3200$

What would happen if we multiplied both the divisor and the dividend by 1,000, 10,000 or 100,000? What do you predict will happen?

How can we use this to help us divide when there are decimals in the divisor? For example, how can we use this to help us divide 172.8 and 3.2?

**Example 1**

Using our discoveries from the discussion, let's divide 537.1 by 8.2.

How can we rewrite this problem using what we learned in Lesson 14?

How could we use the short cut from our discussion to change the original numbers to 5,371 and 82?

**Example 2**

Now let's divide 742.66 by 14.2.

How can we rewrite this division problem so that the divisor is a whole number, but the quotient remains the same?

**Exercises**

Students will participate in a game called Pass the Paper. Students will work in groups of no more than four. There will be a different paper for each player. When the game starts, each student solves the first problem on his paper and passes the paper clockwise to the second student, who uses multiplication to check the work that was done by the previous student. Then, the paper is passed clockwise again to the third student, who solves the second problem. The paper is then passed to the fourth student, who checks the second problem. This process continues until all of the questions on every paper are complete or time runs out.

**Problem Set**

1.  $118.4 \div 6.4$
2.  $314.944 \div 3.7$
3.  $1,840.5072 \div 23.56$



## Lesson 16: Even and Odd Numbers

### Classwork

#### Opening Exercise

What is an even number?

List some examples of even numbers.

What is an odd number?

List some examples of odd numbers.

What happens when we add two even numbers? Will we always get an even number?

**Exercises 1–3**

1. Why is the sum of two even numbers even?
  - a. Think of the problem  $12 + 14$ . Draw dots to represent each number.
  
  
  
  
  
  
  
  
  
  
  - b. Circle pairs of dots to determine if any of the dots are left over.
  
  
  
  
  
  
  
  
  
  
  - c. Will this be true every time two even numbers are added together? Why or why not?
  
  
  
  
  
  
  
  
  
  
2. Why is the sum of two odd numbers even?
  - a. Think of the problem  $11 + 15$ . Draw dots to represent each number.
  
  
  
  
  
  
  
  
  
  
  - b. Circle pairs of dots to determine if any of the dots are left over.
  
  
  
  
  
  
  
  
  
  
  - c. Will this be true every time two odd numbers are added together? Why or why not?

3. Why is the sum of an even number and an odd number odd?
- Think of the problem  $14 + 11$ . Draw dots to represent each number.
  - Circle pairs of dots to determine if any of the dots are left over.
  - Will this be true every time an even number and an odd number are added together? Why or why not?
  - What if the first addend was odd and the second was even? Would the sum still be odd? Why or why not? For example, if we had  $11 + 14$ , would the sum be odd?

Let's sum it up:

- 
- 
-



**Lesson Summary**

Adding:

- The sum of two even numbers is even.
- The sum of two odd numbers is odd.
- The sum of an even number and an odd number is odd.

Multiplying:

- The product of two even numbers is even.
- The product of two odd numbers is odd.
- The product of an even number and an odd number is even.

**Problem Set**

Without solving, tell whether each sum or product is even or odd. Explain your reasoning.

1.  $346 + 721$
2.  $4,690 \times 141$
3.  $1,462,891 \times 745,629$
4.  $425,922 + 32,481,064$
5.  $32 + 45 + 67 + 91 + 34 + 56$



## Lesson 17: Divisibility Tests for 3 and 9

### Classwork

#### Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. You will place some numbers in more than one circle. For example, if 32 were on the list, you would place it in the circles with 2, 4, and 8 because they are all factors of 32.

24; 36; 80; 115; 214; 360; 975; 4,678; 29,785; 414,940

The diagram consists of five large circles arranged in two rows. The top row contains three circles, each with a small square box at the top containing a number: 2, 4, and 5. The bottom row contains two circles, each with a small square box at the top containing a number: 8 and 10. The circles are intended for students to place numbers from the list above based on divisibility.





- e. Is the number 378 divisible by 3? Why or why not?

**Example 2**

Is 3,822 divisible by 3 or 9? Why or why not?

**Exercises 1–5**

Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. Is 2,838 divisible by

3

9

4

Explain your reasoning for your choices.

2. Is 34,515 divisible by

3

9

5

Explain your reasoning for your choices.

3. Is 10,534,341 divisible by

3

9

2

Explain your reasoning for your choices.

4. Is 4,320 divisible by

3

9

10

Explain your reasoning for your choices.

5. Is 6,240 divisible by

3

9

8

Explain your reasoning for your choices.

**Lesson Summary**

To determine if a number is divisible by 3 or 9:

- Calculate the sum of the digits.
- If the sum of the digits is divisible by 3, the entire number is divisible by 3.
- If the sum of the digits is divisible by 9, the entire number is divisible by 9.

Note: If a number is divisible by 9, the number is also divisible by 3.

**Problem Set**

1. Is 32,643 divisible by both 3 and 9? Why or why not?
2. Circle all the factors of 424,380 from the list below.  
2    3    4    5    8    9    10
3. Circle all the factors of 322,875 from the list below.  
2    3    4    5    8    9    10
4. Write a 3 digit number that is divisible by both 3 and 4. Explain how you know this number is divisible by 3 and 4.
5. Write a 4 digit number that is divisible by both 5 and 9. Explain how you know this number is divisible by 5 and 9.

## Lesson 18: Least Common Multiple and Greatest Common Factor

### Classwork

#### Opening

The **Greatest Common Factor** of two whole numbers  $a$  and  $b$ , written  $GCF(a, b)$ , is the greatest whole number, which is a factor of both  $a$  and  $b$ .

The **Least Common Multiple** of two nonzero numbers  $a$  and  $b$ , written  $LCM(a, b)$ , is the least whole number (larger than zero), which is a multiple of both  $a$  and  $b$ .

#### Example 1: Greatest Common Factor

Find the greatest common factor of 12 and 18.

- Listing these factor pairs in order can help you not miss any. Start with one times the number.
- Circle all factors that appear on both lists.
- Place a triangle around the greatest of these common factors.

GCF (12, 18)

12


18


**Example 2: Least Common Multiple**

Find the least common multiple of 12 and 18.

LCM (12, 18)

Write the first 10 multiples of 12.

Write the first 10 multiples of 18.

Circle the multiples that appear on both lists.

Put a rectangle around the least of these common multiples.

**Exercises****Station 1: Factors and GCF**

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

GCF (30, 50)

GCF (30, 45)

GCF (45, 60)

GCF (42, 70)

GCF (96, 144)







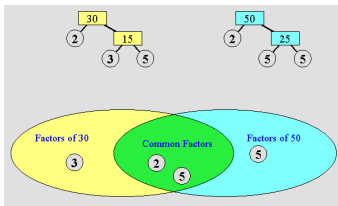
d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

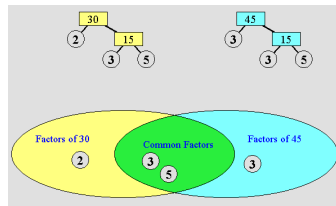
**Station 3: Using Prime Factors to Determine GCF**

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

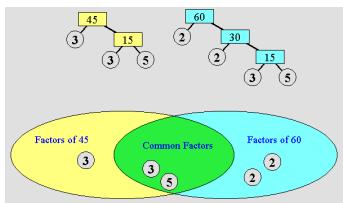
GCF (30, 50)



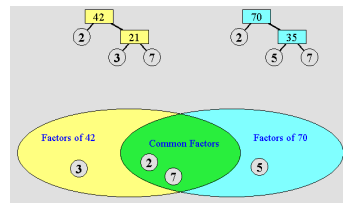
GCF (30, 45)



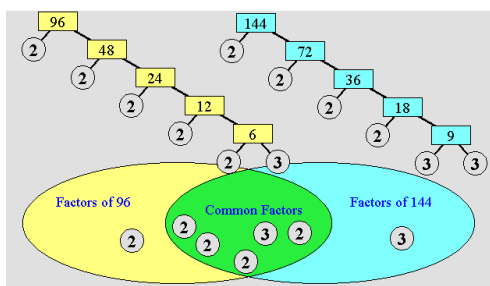
GCF (45, 60)



GCF (42, 70)



GCF (96, 144)





Next, add another new example to one of these two statements applying factors to the distributive property.

Choose any numbers for  $n$ ,  $a$ , and  $b$ .

$$n(a) + n(b) = n(a + b)$$

$$n(a) - n(b) = n(a - b)$$

### Problem Set

Complete the remaining stations from class.



## Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

### Classwork

#### Opening Exercise

Euclid's Algorithm is used to find the greatest common factor (GCF) of two whole numbers.

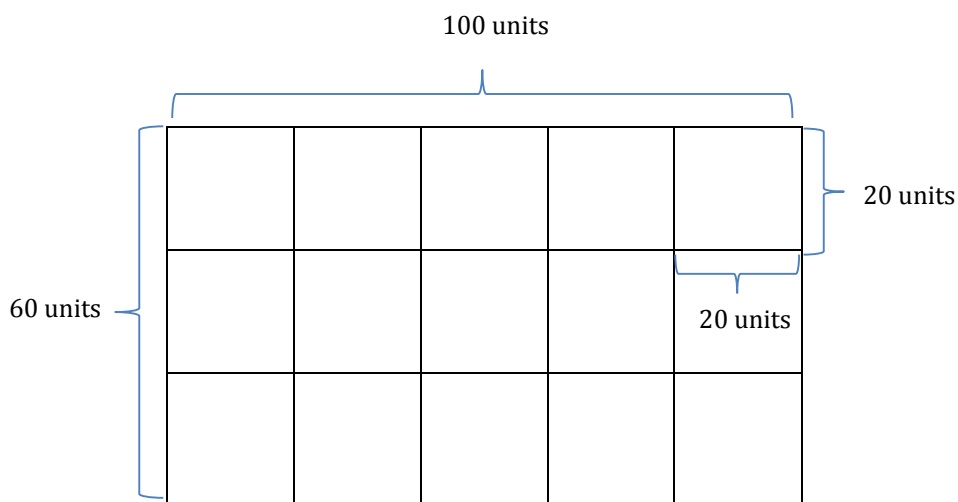
1. Divide the larger of the two numbers by the smaller one.
2. If there is a remainder, divide it into the divisor.
3. Continue dividing the last divisor by the last remainder until the remainder is zero.
4. The final divisor is the GCF of the original pair of numbers.

$$383 \div 4 =$$

$$432 \div 12 =$$

$$403 \div 13 =$$

#### Example 1: Euclid's Algorithm Conceptualized





**Example 4: Area Problems**

The greatest common factor has many uses. Among them, the GCF lets us find out the maximum size of squares that will cover a rectangle. When we solve problems like this, we cannot have any gaps or any overlapping squares. Of course, the maximum size squares will be the minimum number of squares needed.

A rectangular computer table measures 30 inches by 50 inches. We need to cover it with square tiles. What is the side length of the largest square tile we can use to completely cover the table so that there is no overlap or gaps?



- a. If we use squares that are 10 by 10, how many will we need?
  
  
  
  
  
  
  
  
  
  
- b. If this were a giant chunk of cheese in a factory, would it change the thinking or the calculations we just did?
  
  
  
  
  
  
  
  
  
  
- c. How many 10 inch  $\times$  10 inch squares of cheese could be cut from the giant 30 inch  $\times$  50 inch slab?

**Problem Set**

1. Use Euclid's Algorithm to find the greatest common factor of the following pairs of numbers:
  - a. GCF (12,78)
  - b. GCF (18,176)
  
2. Juanita and Samuel are planning a pizza party. They order a rectangular sheet pizza which measures 21 inches by 36 inches. They tell the pizza maker not to cut it because they want to cut it themselves.
  - a. All pieces of pizza must be square with none left over. What is the length of the side of the largest square pieces into which Juanita and Samuel can cut the pizza?
  - b. How many pieces of this size will there be?
  
3. Shelly and Mickelle are making a quilt. They have a piece of fabric that measures 48 inches by 168 inches.
  - a. All pieces of fabric must be square with none left over. What is the length of the side of the largest square pieces into which Shelly and Mickelle can cut the fabric?
  - b. How many pieces of this size will there be?



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## Lesson 1: Interpreting Division of a Fraction by a Whole

### Number—Visual Models

#### Exit Ticket

Find the quotient using a model.

1.  $\frac{2}{3} \div 3$

2.  $\frac{5}{6} \div 2$



Fraction cards to use at the beginning of class:

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{5}$	$\frac{2}{6}$
$\frac{1}{3}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$
$\frac{7}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{3}{8}$



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## Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

### Exit Ticket

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

2. Rachel completed  $\frac{3}{4}$  of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?



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## Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

### Exit Ticket

Draw a model to support your answer to the division questions.

1.  $\frac{9}{4} \div \frac{3}{4}$

2.  $\frac{7}{3} \div \frac{2}{3}$





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## Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

### Exit Ticket

Draw a model to support your answer to the division questions.

1.  $\frac{9}{4} \div \frac{3}{8}$

2.  $\frac{3}{5} \div \frac{2}{3}$



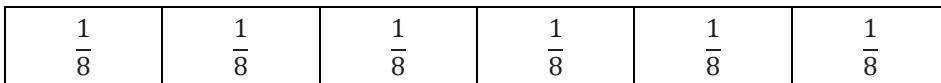
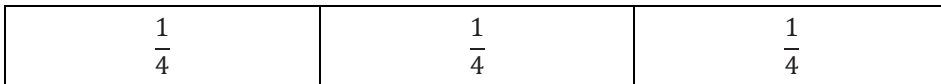
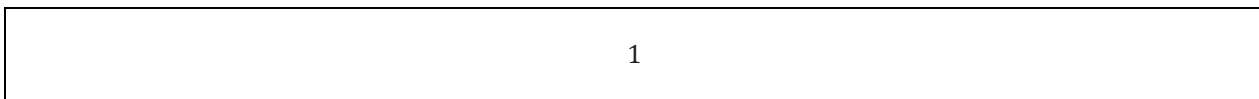
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## Lesson 5: Creating Division Stories

### Exit Ticket

Write a story problem for the following measurement division:  $\frac{3}{4} \div \frac{1}{8} = 6$ .





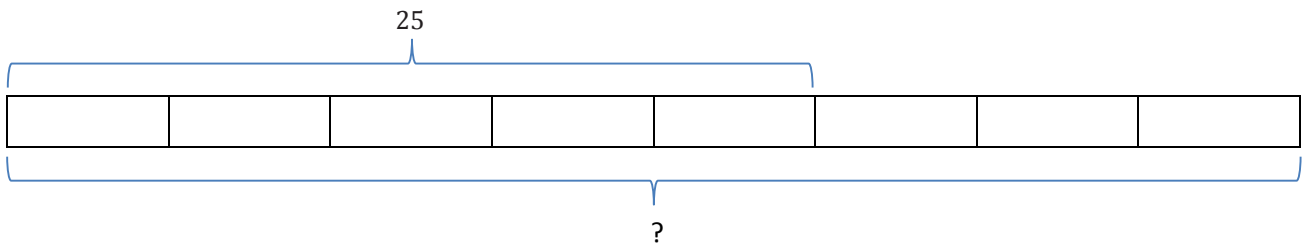
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## Lesson 6: More Division Stories

### Exit Ticket

Write a word problem for the following partitive division:  $25 \div \frac{5}{8} = 40$ .





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## Lesson 7: The Relationship Between Visual Fraction Models and Equations

### Exit Ticket

1. Write the reciprocal of the following numbers.

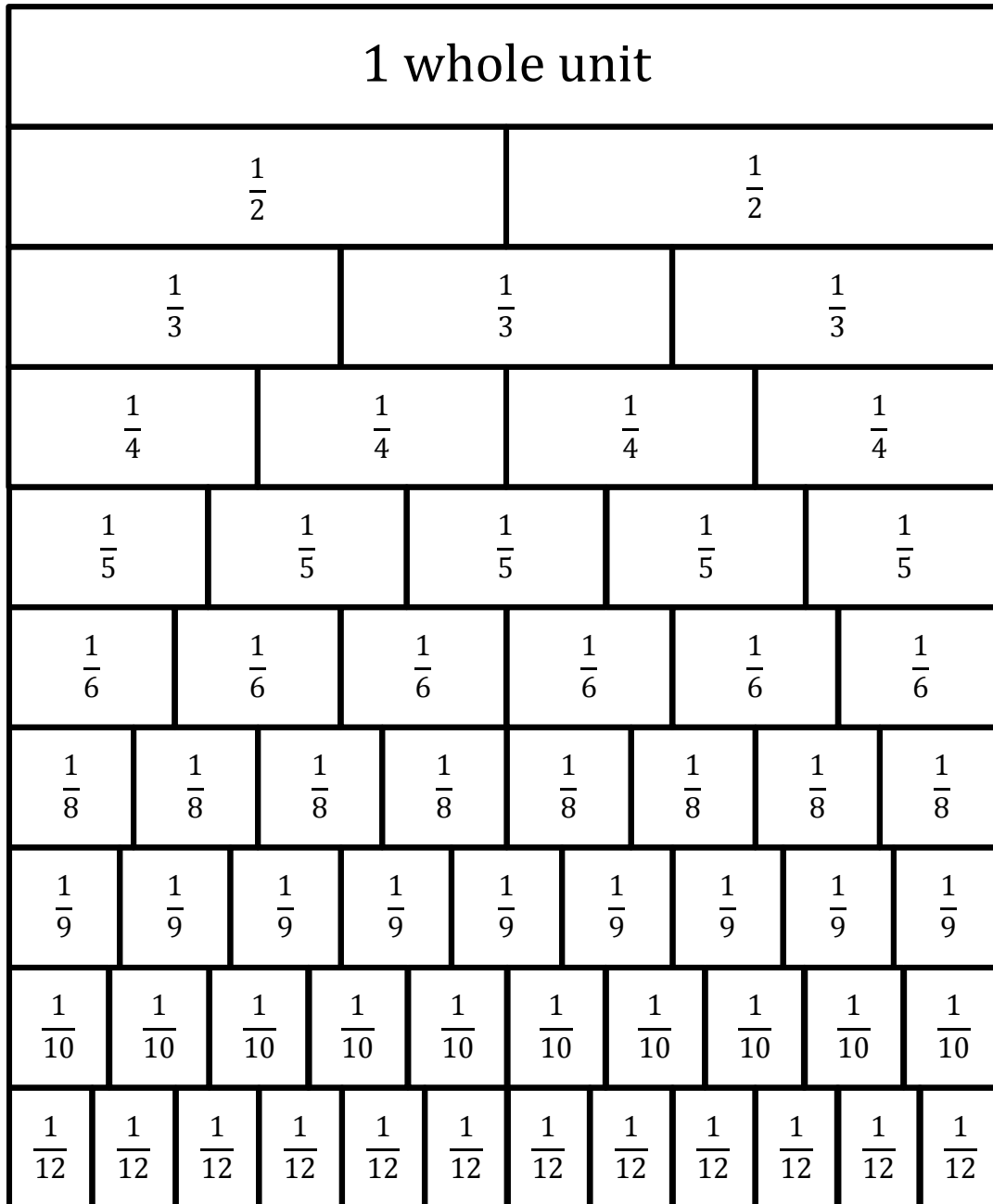
Number	$\frac{7}{10}$	$\frac{1}{2}$	5
Reciprocal			

2. Rewrite this division problem as a multiplication problem:  $\frac{5}{8} \div \frac{2}{3}$ .

3. Solve Problem 2 using models.









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## Lesson 8: Dividing Fractions and Mixed Numbers

### Exit Ticket

Calculate the quotient.

1.  $\frac{3}{4} \div 5\frac{1}{5}$

2.  $\frac{3}{7} \div 2\frac{1}{2}$

3.  $\frac{5}{8} \div 6\frac{5}{6}$

4.  $\frac{5}{8} \div 8\frac{3}{10}$



Memory Game

A. $\frac{3}{4} \div 6\frac{2}{3}$	$\frac{9}{80}$	B. $\frac{1}{3} \div 4\frac{3}{4}$	$\frac{4}{57}$
C. $\frac{2}{5} \div 1\frac{7}{8}$	$\frac{16}{75}$	D. $7\frac{1}{2} \div \frac{5}{6}$	9
E. $3\frac{4}{7} \div \frac{5}{8}$	$5\frac{5}{7}$	F. $5\frac{5}{8} \div \frac{9}{10}$	$6\frac{1}{4}$
G. $\frac{1}{4} \div 10\frac{11}{12}$	$\frac{3}{131}$	H. $5\frac{3}{4} \div \frac{5}{9}$	$10\frac{7}{20}$
I. $3\frac{1}{5} \div \frac{2}{3}$	$5\frac{4}{5}$	J. $\frac{3}{5} \div 3\frac{1}{7}$	$\frac{21}{110}$
K. $\frac{10}{13} \div 2\frac{4}{7}$	$\frac{35}{117}$	L. $2\frac{1}{4} \div \frac{7}{8}$	$2\frac{4}{7}$



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## Lesson 9: Sums and Differences of Decimals

### Exit Ticket

Solve each problem. Show that the placement of the decimal is correct through either estimation or fraction calculation.

1.  $382\frac{3}{10} - 191\frac{87}{100}$

2.  $594\frac{7}{25} + 89\frac{37}{100}$





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## Lesson 10: The Distributive Property and the Products of Decimals

### Exit Ticket

Complete the problem using partial products.

$$500 \times 12.7$$







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## Lesson 12: Estimating Digits in a Quotient

### Exit Ticket

1. Estimate the quotient:  $1,908 \div 36$ .
  
  
  
  
  
  
  
  
  
  
2. Use the division algorithm and your estimate to find the quotient:  $1,908 \div 36$ .
  
  
  
  
  
  
  
  
  
  
3. Use estimation to determine if  $8,580 \div 78$  has a quotient in the 10s, 100s, or 1000s.



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## Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

### Exit Ticket

Divide using the division algorithm:  $392,196 \div 87$ .









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## Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

### Exit Ticket

State the power of 10 you would use to convert the given decimal division to whole number division. Then, complete the multiplication on the dividend and divisor.

1.  $133.84 \div 5.6$

2.  $12.4 \div 1.036$

3.  $38.9 \div 2.91$

4.  $45 \div 1.5$



## Player A

1. $15.5 \div 6.2$	Check:
2. $28.08 \div 7.8$	Check:
3. $44.888 \div 3.62$	Check:
4. $3,912.99 \div 15.9$	Check:
5. $865.1475 \div 47.25$	Check:



## Player B

1. $32.4 \div 7.2$	Check:
2. $49.14 \div 6.3$	Check:
3. $39.321 \div 2.57$	Check:
4. $8,578.02 \div 24.6$	Check:
5. $439.0464 \div 35.18$	Check:





# Player C

1. $25.9 \div 7.4$	Check:
2. $25.48 \div 5.2$	Check:
3. $61.962 \div 4.49$	Check:
4. $16,437.42 \div 31.8$	Check:
5. $1,238.8048 \div 52.76$	Check:



## Player D

1. $63.7 \div 9.8$	Check:
2. $32.68 \div 8.6$	Check:
3. $142.912 \div 8.12$	Check:
4. $23,344.58 \div 57.4$	Check:
5. $2,498.743 \div 39.65$	Check:



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## Lesson 16: Even and Odd Numbers

### Exit Ticket

Determine whether each sum or product will be even or odd. Explain your reasoning.

1.  $56,426 + 17,895$

2.  $317,362 \times 129,324$

3.  $10,481 + 4,569$

4.  $32,457 \times 12,781$

5. Show or explain why  $12 + 13 + 14 + 15 + 16$  will result in an even sum.









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## Lesson 18: Least Common Multiple and Greatest Common Factor

### Exit Ticket

- Find the LCM and GCF of 12 and 15.
- Write two numbers, neither of which is 8, whose GCF is 8.
- Write two numbers, neither of which is 28, whose LCM is 28.

Rate each of the stations you visited today. Use this scale:

3—Easy—I've got it; I don't need any help.

2 —Medium—I need more practice, but I understand some of it.

1—Hard—I'm not getting this yet.

Complete the following chart:

Station	Rating (3, 2, 1)	Comment to the Teacher
Station 1: Factors and GCF		
Station 2: Multiples and LCM		
Station 3: Using Prime Factors for GCF		
Station 4: Applying Factors to the Distributive Property		





**Station 2: Multiples and LCM**

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

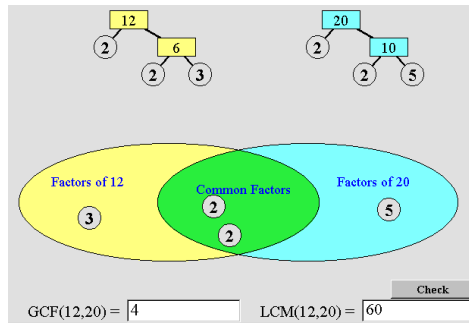
Find the least common multiple of one of these pairs: 9, 12; 8, 18; 4, 30; 12, 30; 20, 50.

Next, choose one of these problems that has not yet been solved:

- a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?
- b. Starting at 6:00 a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?
- c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?
- d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.
- e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Solve it together on your student page. Then, use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

**Station 3: Using Prime Factors to Determine GCF**



Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Use Prime Factors to find the Greatest Common Factor of one of the following pairs of numbers:

- 30, 50    30, 45    45, 60    42, 70    96, 144

Next, choose one of these problems that has not yet been solved:

- a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?
  
- b. Find the GCF of your original pair of numbers.
  
- c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?
  
- d. Glenn’s favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat, and all of the prime numbers are less than 12. What is Glenn’s number? When was Sarah born?

**Station 4: Applying Factors to the Distributive Property**

Study these examples of how factors apply to the distributive property.

$$8 + 12 = 4(2) + 4(3) = 4(2 + 3) = 20$$

$$4(2) + 4(3) = 4(5) = 20$$

$$15 + 25 = 5(3) + 5(5) = 5(3 + 5) = 40$$

$$5(3) + 5(5) = 5(8) = 40$$

$$36 - 24 = 4(9) - 4(6) = 4(9 - 6) = 12$$

$$4(9) - 4(6) = 4(3) = 12$$

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Find the GCF from the two numbers, and rewrite the sum using the distributive property.

1.  $12 + 18 =$

2.  $42 + 14 =$

3.  $36 + 27 =$

4.  $16 + 72 =$

5.  $44 + 33 =$

Next, add another new example to one of these two statements applying factors to the distributive property.

Choose any numbers for  $n$ ,  $a$ , and  $b$ .

$$n(a) + n(b) = n(a + b)$$

$$n(a) - n(b) = n(a - b)$$

Name \_\_\_\_\_

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## Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

### Exit Ticket

Use Euclid's Algorithm to find the greatest common factor of 45 and 75.

